

# Lattice QCD with two flavor dynamical domain-wall quarks

Taku Izubuchi (BNL HET)

for Riken-BNL-Columbia collaboration

# Contents

1. Motivations
2. domain-wall fermion
3.  $\chi$  symmetry breaking and Spectrum Flow
4. Simulation
5. Results
6. Discussions

# Motivation for lattice $\chi$ fermions

chiral symmetry

$$\begin{aligned}\mathcal{L} &= \bar{q} \not{D} q \\ q &\rightarrow e^{i\alpha\gamma_5} q, \quad \bar{q} \rightarrow \bar{q} e^{i\alpha\gamma_5} \\ \{\not{D}, \gamma_5\} &= 0\end{aligned}$$

Lattice Gauge theory: a **Gauge symmetric** Regularization

Physics beyond perturbative analysis

- Hadron Physics,
  - Structure functions/Form factors,
  - $\chi$ -PT *e.g.*  $\epsilon'/\epsilon$
  - physics around light pions
- chiral Anomaly & Topology ,  $\eta'$
- Finite temperature/density,  $T \neq 0$  ,  $\mu \neq 0$
- Super Yang-Mills

(esp. useful for **physics with chiral symmetry**)

Fine tuning of fermion mass is needless

c.f. light Wilson Fermion  $\delta M_W = -cg^2$ , exceptional configs

scaling violation should be small.( lattice spacing,  $a > 0$ )

No dim=5 operator with  $\chi$ -sym.

$$\delta(\mathcal{L}) \sim a^2 O_6 + \dots$$

Fine tuning of fermion mass is needless

c.f. light Wilson Fermion  $\delta M_W = -cg^2$ , exceptional configs

scaling violation should be small. ( lattice spacing,  $a > 0$ )

No dim=5 operator with  $\chi$ -sym.

$$\delta(\mathcal{L}) \sim a^2 O_6 + \dots \quad (\text{for } L_s \rightarrow \infty)$$

Fine tuning of fermion mass is needless

c.f. light Wilson Fermion  $\delta M_W = -cg^2$ , exceptional configs

scaling violation should be small. ( lattice spacing,  $a > 0$ )

No dim=5 operator with  $\chi$ -sym.

$$\delta(\mathcal{L}) \sim a^2 O_6 + \dots \quad (\text{for } L_s \rightarrow \infty)$$

Could use relatively large  $a$  to predict continuum limit

unphysical mixing is prohibited by  $\chi$ -sym

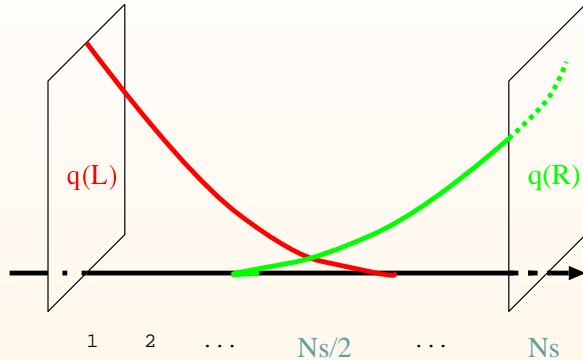
c.f. Wilson/Clover  $\mathbb{Z} O_V^{con} = O_V + O(a)O_A$

(in  $L_s \rightarrow \infty$ ) automatic **on/off-shell  $O(a)$  improved fermion**

$$S_{con} = S_{DWF} + O(a^2)$$

$$O_{con} = ZO_{DWF} + O(a^2)$$

# Domain-Wall Fermions



$$\psi(x, \textcolor{red}{s})$$

▷  $x, y$  : space-time  $\textcolor{blue}{s}, \textcolor{red}{t}$  : 5th coordinate (species)

$$D_{DWF}\psi(x, \textcolor{red}{s}) = -P_+\psi(x, \textcolor{red}{s}-1) + (1 + D_W)\psi(x, \textcolor{red}{s}) - P_-\psi(x, \textcolor{red}{s}+1)$$

$$D_W(x, y) = (4 - M) - \sum_\mu \frac{1}{2} \left[ (1 - \gamma_\mu) U_{x,\mu} \delta_{x+\hat{\mu},y} + (1 + \gamma_\mu) U_{y,\mu}^\dagger \delta_{x-\hat{\mu},y} \right],$$

$$P_\mp = \frac{1}{2}(1 \mp \gamma_5)$$

4-dim quarks

$$q(x) = P_- \psi(x, 1) + P_+ \psi(x, L_s)$$

$$\bar{q}(x) = \bar{\psi}(x, L_s) P_- + \bar{\psi}(x, 1) P_+$$

## Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

## Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

$$-P_+\psi(x, \textcolor{red}{s-1}) + (1 + D_W)\psi(x, \textcolor{red}{s}) - P_-\psi(x, \textcolor{red}{s+1}) = 0$$

## Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

$$\begin{aligned} -P_+\psi(x, \textcolor{red}{s-1}) &+ (1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s+1}) &= 0 \\ -P_+\psi(x, \textcolor{red}{s-1}) &+ P_+(1 + D_W)\psi(x, \textcolor{red}{s}) & = 0 \end{aligned}$$

# Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

$$\begin{aligned} -P_+\psi(x, \textcolor{red}{s-1}) &+ (1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s+1}) &= 0 \\ -P_+\psi(x, \textcolor{red}{s-1}) &+ P_+(1 + D_W)\psi(x, \textcolor{red}{s}) & = 0 \\ &+ P_-(1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s+1}) &= 0 \end{aligned}$$

# Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

$$\begin{aligned} -P_+\psi(x, \textcolor{red}{s-1}) &+ (1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s+1}) &= 0 \\ -P_+\psi(x, \textcolor{red}{s-1}) &+ P_+(1 + D_W)\psi(x, \textcolor{red}{s}) & = 0 \\ &+ P_-(1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s+1}) &= 0 \end{aligned}$$

a recursive relation between  $\textcolor{red}{s-1}$  and  $\textcolor{red}{s}$

$$P_+\psi(x, \textcolor{red}{s-1}) = P_+(1 + D_W)\psi(x, \textcolor{red}{s}) = 0$$

$$P_-(1 + D_W)\psi(x, \textcolor{red}{s-1}) = P_-\psi(x, \textcolor{red}{s}) = 0$$

# Dirac Equation of DWF

Dirac equation, or 0-mode

$$D_{DWF}\psi(x, \textcolor{red}{s}) = 0$$

For generic  $\textcolor{red}{s}$  ( $s \neq 1, L_s$ )

$$\begin{aligned} -P_+\psi(x, \textcolor{red}{s}-1) &+ (1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s}+1) = 0 \\ -P_+\psi(x, \textcolor{red}{s}-1) &+ P_+(1 + D_W)\psi(x, \textcolor{red}{s}) &= 0 \\ &+ P_-(1 + D_W)\psi(x, \textcolor{red}{s}) & -P_-\psi(x, \textcolor{red}{s}+1) = 0 \end{aligned}$$

a recursive relation between  $\textcolor{red}{s}-1$  and  $\textcolor{red}{s}$

$$P_+\psi(x, \textcolor{red}{s}-1) = P_+(1 + D_W)\psi(x, \textcolor{red}{s}) = 0$$

$$P_-(1 + D_W)\psi(x, \textcolor{red}{s}-1) = P_-\psi(x, \textcolor{red}{s}) = 0$$

$$[P_+ + P_-(1 + D_W)]\psi(x, \textcolor{red}{s}-1) = [P_- + P_+(1 + D_W)]\psi(x, \textcolor{red}{s})$$

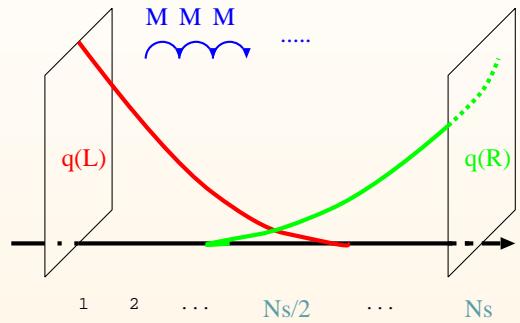
translation matrix  $M$  and  $H_W = \gamma_5 D_W$ ,

$$\begin{aligned}
\psi(x, s) &= M\psi(x, s-1) , \\
M &= [P_- + P_+(1 + D_W)]^{-1} [P_+ + P_-(1 + D_W)] \\
&= [(2 + D_W) + \gamma_5 D_W]^{-1} \textcolor{red}{1} [(2 + D_W) - \gamma_5 D_W] \\
&= [(2 + D_W) + \textcolor{blue}{H}_W]^{-1} \textcolor{red}{(2 + D_W)^{-1}} (2 + D_W) [(2 + D_W) - \textcolor{blue}{H}_W] \\
&= [1 + (2 + D_W)^{-1} H_W]^{-1} [1 - (2 + D_W)^{-1} H_W] \\
&= \frac{1 - \textcolor{red}{M}_T}{1 + \textcolor{red}{M}_T}
\end{aligned}$$

$M_T$

$$\begin{aligned}
M_T &= \frac{1}{2 + D_W} H_W \\
&\sim H_W \quad (\text{for modes with small eigen value })
\end{aligned}$$

# Zero modes and $\chi$ sym. breaking



## Axial Ward Takahashi identity

$$\partial_\mu \langle A_\mu(x) P(y) \rangle = 2m_f \langle P(x) P(y) \rangle + 2 \langle J_{5q}(x) P(y) \rangle - 2 \langle \bar{q}q(x) \rangle \delta_{x,y}$$

- non-local axial current  $A_\mu(x)$
- explicit breaking operator:  
$$J_{5q}(x) = -\bar{\psi}(x, \frac{L_s}{2}) P_L \psi(x, \frac{L_s}{2} + 1) + \bar{\psi}(x, \frac{L_s}{2} + 1) P_R \psi(x, \frac{L_s}{2}).$$

Approximately near zero eigen value of  $H_W$  controls chiral symmetry breaking

# Dynamical DWF configurations on coarse lattices

(Columbia)

- $8^3 \times 32, \beta = 5.325$  plaq.,  $a_\rho^{-1} = 0.7$  GeV
- $8^3 \times 32, \beta = 1.9/2.0$  Iwasaki,  $a_\rho^{-1} = 0.7/0.8$  GeV

for  $N_F = 2, L_s = 24, M_5 = 1.9$

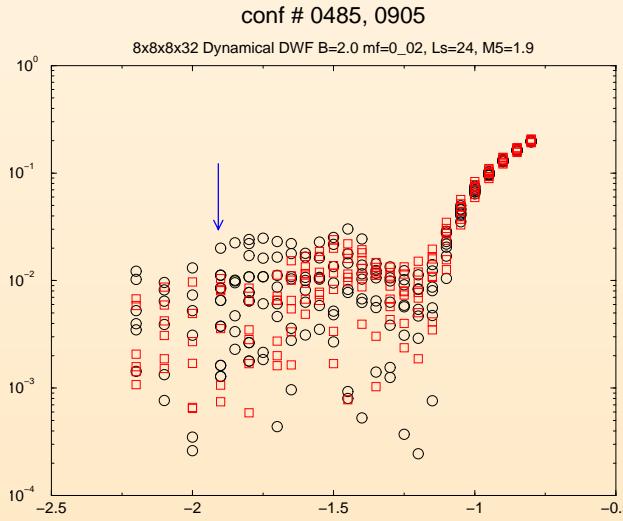
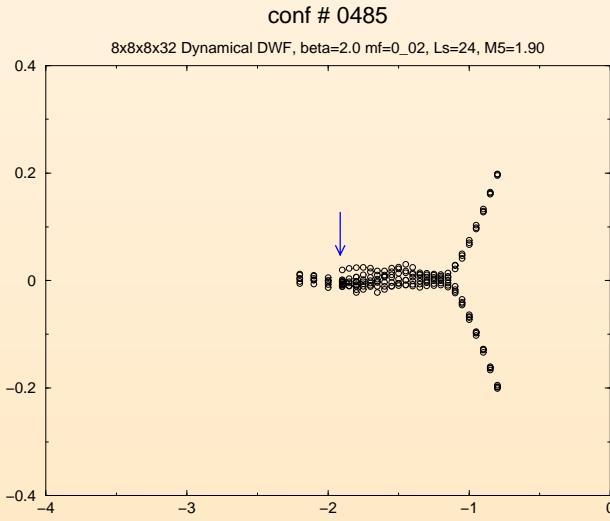
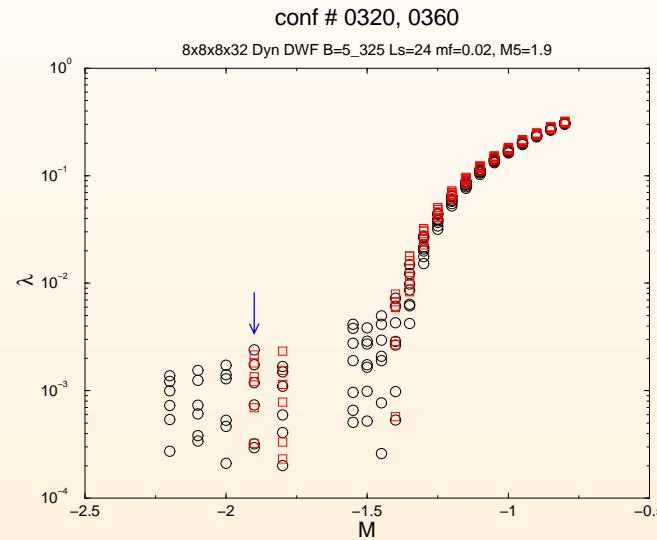
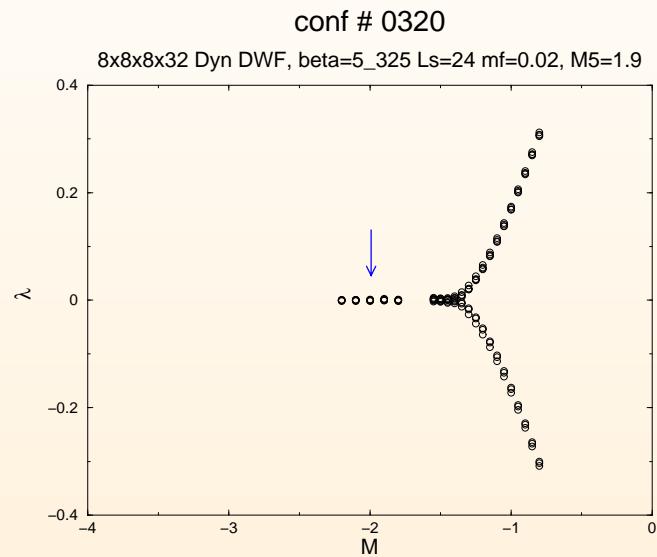
Chiral sym:  $M_\pi(m_f \rightarrow 0) \sim 390/340/260$  MeV

needs  $L_s \sim \overline{100}$  !

$\implies?$   $L_s \sim 10$  at  $a^{-1} \sim 2$  GeV

# Dynamical DWF configuration at coarse lattice

## eigenvalue $H_W$ as a function of $M$



near zero,  $\sim 10^{-3}$ , eigenvalue

$\Rightarrow$  slow damping (No exponential suppression)

$\sim$  Parity Broken Phase :

$\sim$  Anderson localization with rough impurity

$$\langle \bar{q} \gamma_5 q \rangle = -2\pi\rho_W(0) \neq 0$$

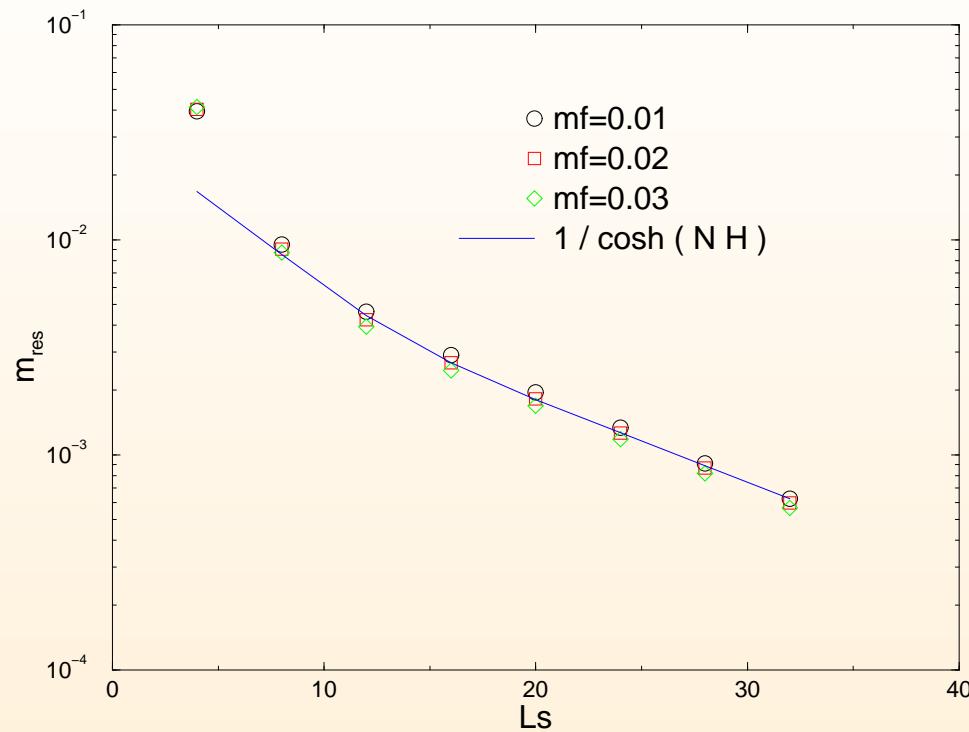
rel. for 4D Effective propagators:

$$\left\langle \psi_{\frac{L_s}{2}} \bar{q} \right\rangle = \frac{1}{\cosh(\tilde{H} L_s / 2)} \langle q \bar{q} \rangle$$

leads exponential suppression:

$$m_{res} = \frac{\langle J_{5q} P \rangle}{\langle PP \rangle} \sim \frac{1}{\cosh^2(\tilde{H} L_s / 2)} \sim \exp(-\textcolor{red}{L}_s \tilde{H})$$

DBW2 B=0.87, 8x8, conf # 7



using DBW2 action

## Dynamical DWF

- $a^{-1} \sim 2GeV$
- using improved glue (DBW2,...)
- scale matching with dynamical staggered fermion on deconfined phase
- New force term

# Dynamical DWF

- $a^{-1} \sim 2GeV$
- using improved glue (DBW2,...)
- scale matching with dynamical staggered fermion on deconfined phase
- New force term

HMC

$$\begin{aligned} Z &= \int DU_\mu \det D_{GW}^\dagger D_{GW}(m_f) e^{-S_g} \\ &= \int DU_\mu \frac{\det D_{DWF}^\dagger D_{DWF}(m_f)}{\det D_{DWF}^\dagger D_{DWF}(1)} e^{-S_g} \\ &= \int D[U_\mu, \Phi_F, \Phi_{PV}, P] \exp \left\{ -\Phi_F^\dagger [D^\dagger D(m_f)]^{-1} \Phi_F - \Phi_{PV}^\dagger D^\dagger D(1) \Phi_{PV} - S_g + P^2/2 \right\} \end{aligned}$$

# Dynamical DWF

- $a^{-1} \sim 2\text{GeV}$
- using improved glue (DBW2,...)
- scale matching with dynamical staggered fermion on deconfined phase
- New force term

HMC

$$\begin{aligned} Z &= \int DU_\mu \det D_{GW}^\dagger D_{GW}(m_f) e^{-S_g} \\ &= \int DU_\mu \frac{\det D_{DWF}^\dagger D_{DWF}(m_f)}{\det D_{DWF}^\dagger D_{DWF}(1)} e^{-S_g} \\ &= \int D[U_\mu, \Phi_F, \Phi_{PV}, P] \exp \left\{ -\Phi_F^\dagger [D^\dagger D(m_f)]^{-1} \Phi_F - \Phi_{PV}^\dagger D^\dagger D(1) \Phi_{PV} - S_g + P^2/2 \right\} \\ &= \int DU_\mu \frac{1}{\det \{ [D^{-1}(m_f) D_{DWF}(1)]^\dagger [D^{-1}(m_f) D_{DWF}(1)] \}} e^{-S_g} \end{aligned}$$

# Dynamical DWF

- $a^{-1} \sim 2GeV$
- using improved glue (DBW2,...)
- scale matching with dynamical staggered fermion on deconfined phase
- New force term

HMC

$$\begin{aligned}
Z &= \int DU_\mu \det D_{GW}^\dagger D_{GW}(m_f) e^{-S_g} \\
&= \int DU_\mu \frac{\det D_{DWF}^\dagger D_{DWF}(m_f)}{\det D_{DWF}^\dagger D_{DWF}(1)} e^{-S_g} \\
&= \int D[U_\mu, \Phi_F, \Phi_{PV}, P] \exp \left\{ -\Phi_F^\dagger [D^\dagger D(m_f)]^{-1} \Phi_F - \Phi_{PV}^\dagger D^\dagger D(1) \Phi_{PV} - S_g + P^2/2 \right\} \\
&= \int DU_\mu \frac{1}{\det \{ [D^{-1}(m_f) D_{DWF}(1)]^\dagger [D^{-1}(m_f) D_{DWF}(1)] \}} e^{-S_g} \\
&= \int D[U_\mu, \Phi_F, P] \exp \left\{ -\Phi_F^\dagger [D^{-1}(m_f) D(1)]^\dagger [\textcolor{red}{D^{-1}(m_f) D(1)}] \Phi_F - S_g + P^2/2 \right\}
\end{aligned}$$

## Dynamical DWF on finer lattices

- target scale :  $a^{-1} \sim 2$  GeV
- Lattice size: (X,Y,Z,T) = (16,8,8,8), (16,16,16,8),ABC for T-dir
- Glueon: WilsonG  $\beta = 5.80$ , DBW2,  $\beta = 0.80$ , Iwasaki ...
- quarks:  $N_F = 2$ ,  $m_f(dyn) = 0.025, 0.020$
- DWF param.:  $M = 1.80, 1.70$ ,  $L_s = 8, 12, (16)$
- HMC param.:  $\tau=0.5$ ,  $\Delta\tau = 0.5/\{25, 32, 50\}$

# Dynamical DWF on finer lattices

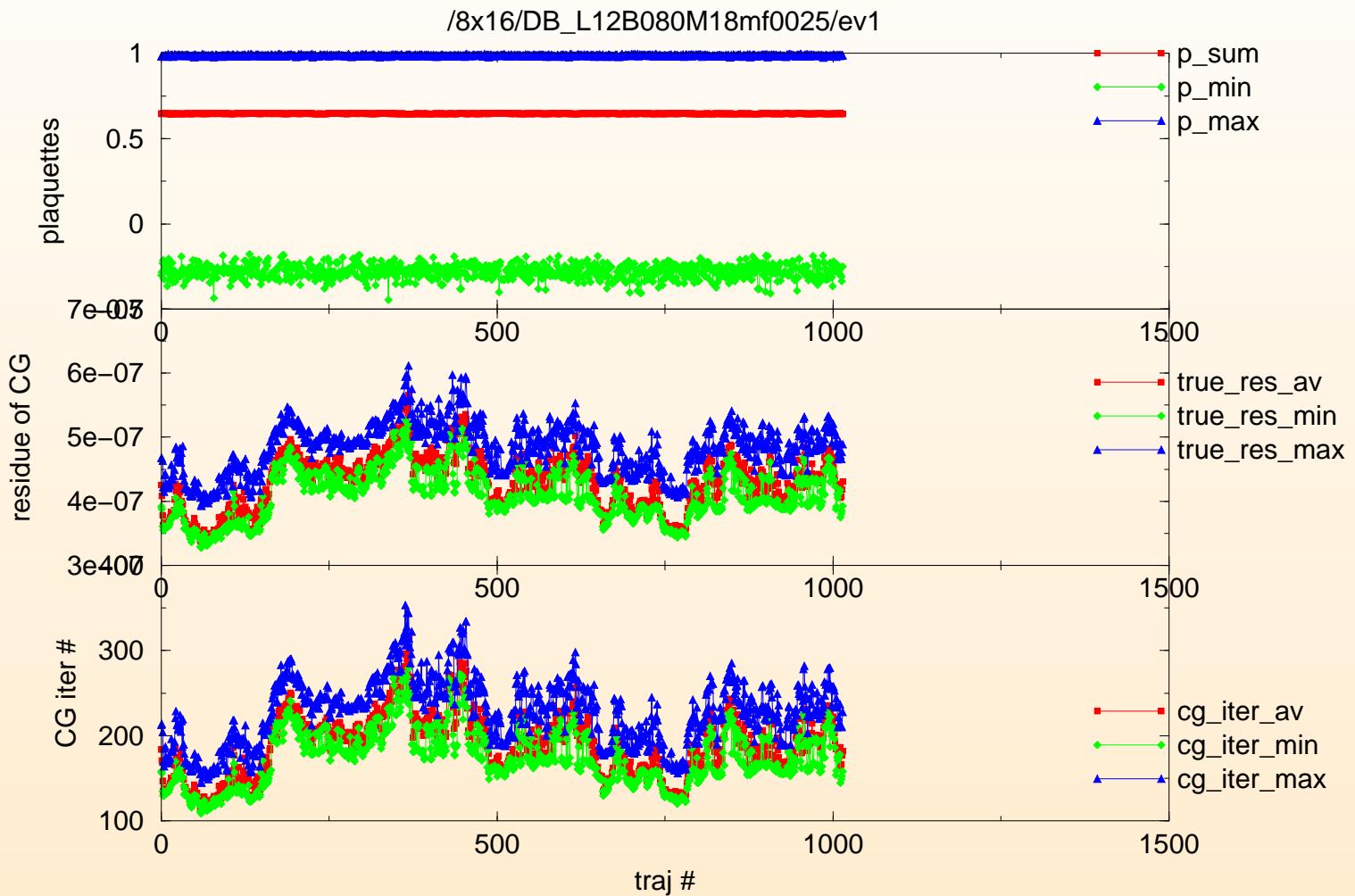
- target scale :  $a^{-1} \sim 2$  GeV
- Lattice size: (X,Y,Z,T) = (16,8,8,8), (16,16,16,8),ABC for T-dir
- Glueon: WilsonG  $\beta = 5.80$ , DBW2,  $\beta = 0.80$ , Iwasaki ...
- quarks:  $N_F = 2$ ,  $m_f(dyn) = 0.025, 0.020$
- DWF param.:  $M = 1.80, 1.70$ ,  $L_s = 8, 12, (16)$
- HMC param.:  $\tau=0.5$ ,  $\Delta\tau = 0.5/\{25, 32, 50\}$

## • Preliminary

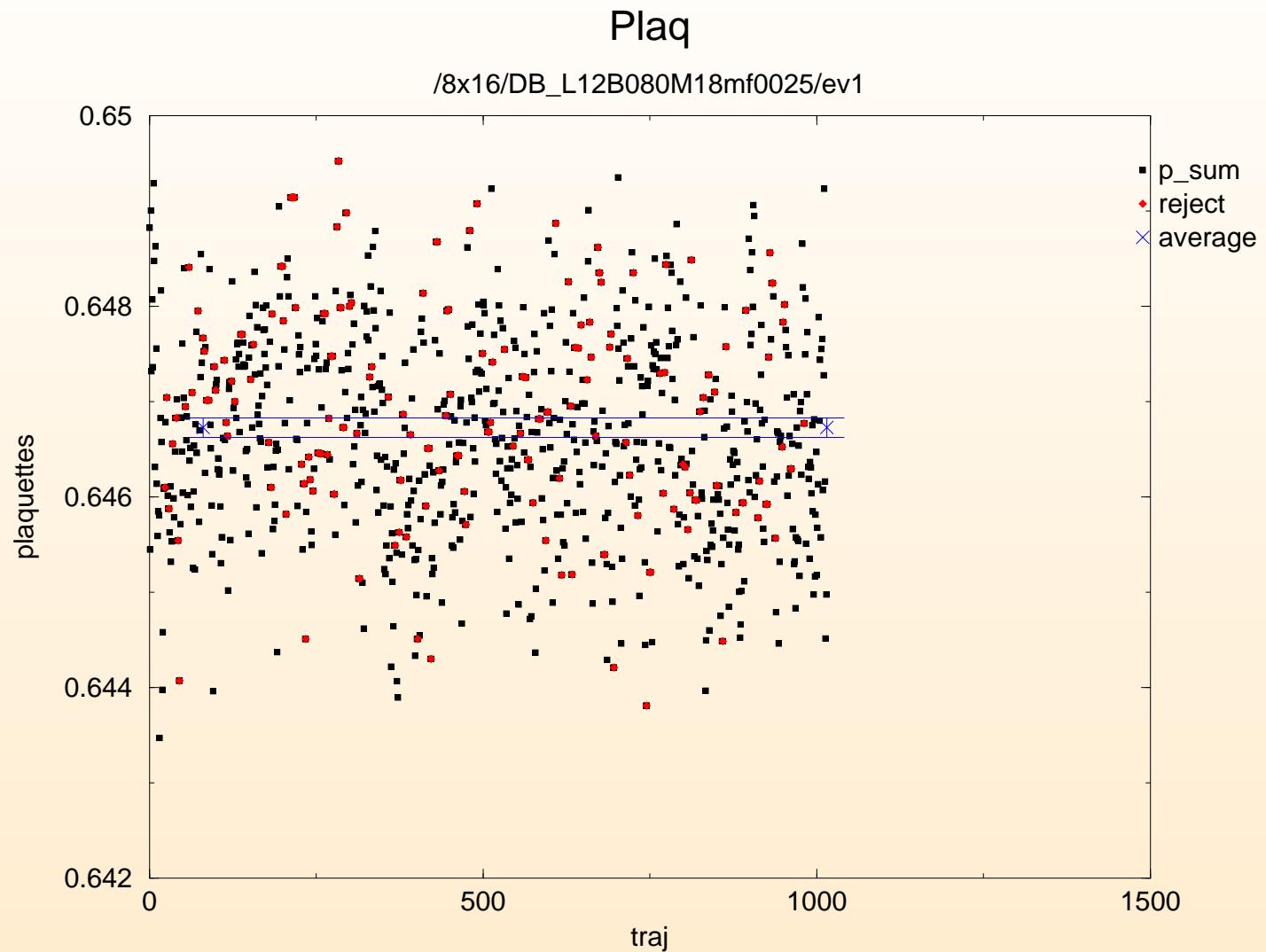
- num CG.  $\sim 100--300$ .
- thermalization after  $\sim 600--1000$  trajs.
- fewer CG for the new force term
- acceptance  $\sim 70\% -- 95\%$
- elapse time for  $8^3 \times 16 \sim$  (an hour / 10 traj for)

# HMC basics history

## HMC basics history



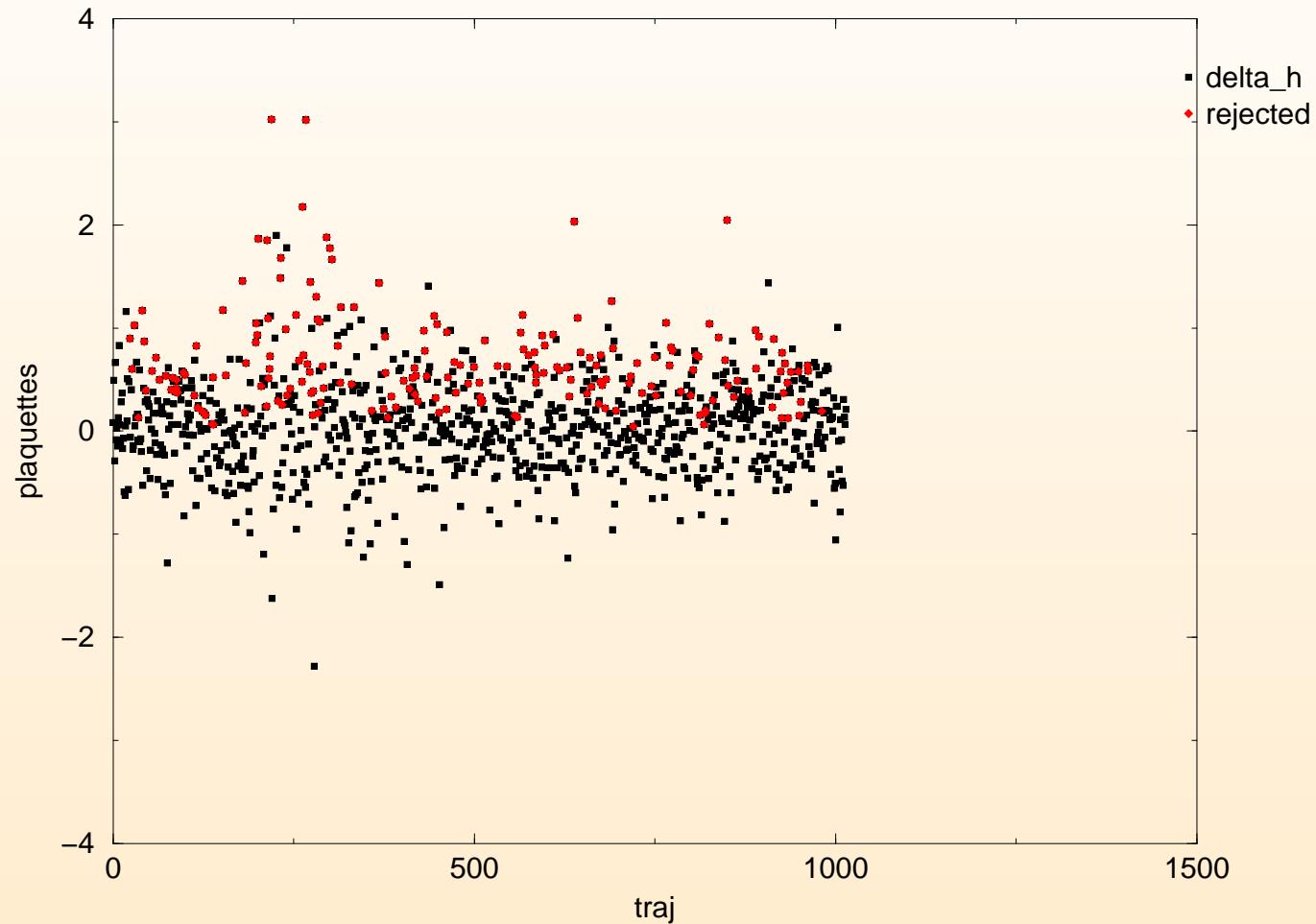
# Plaquette history



# $\Delta$ H history

## Delta H

/8x16/DB\_L12B080M18mf0025/ev1

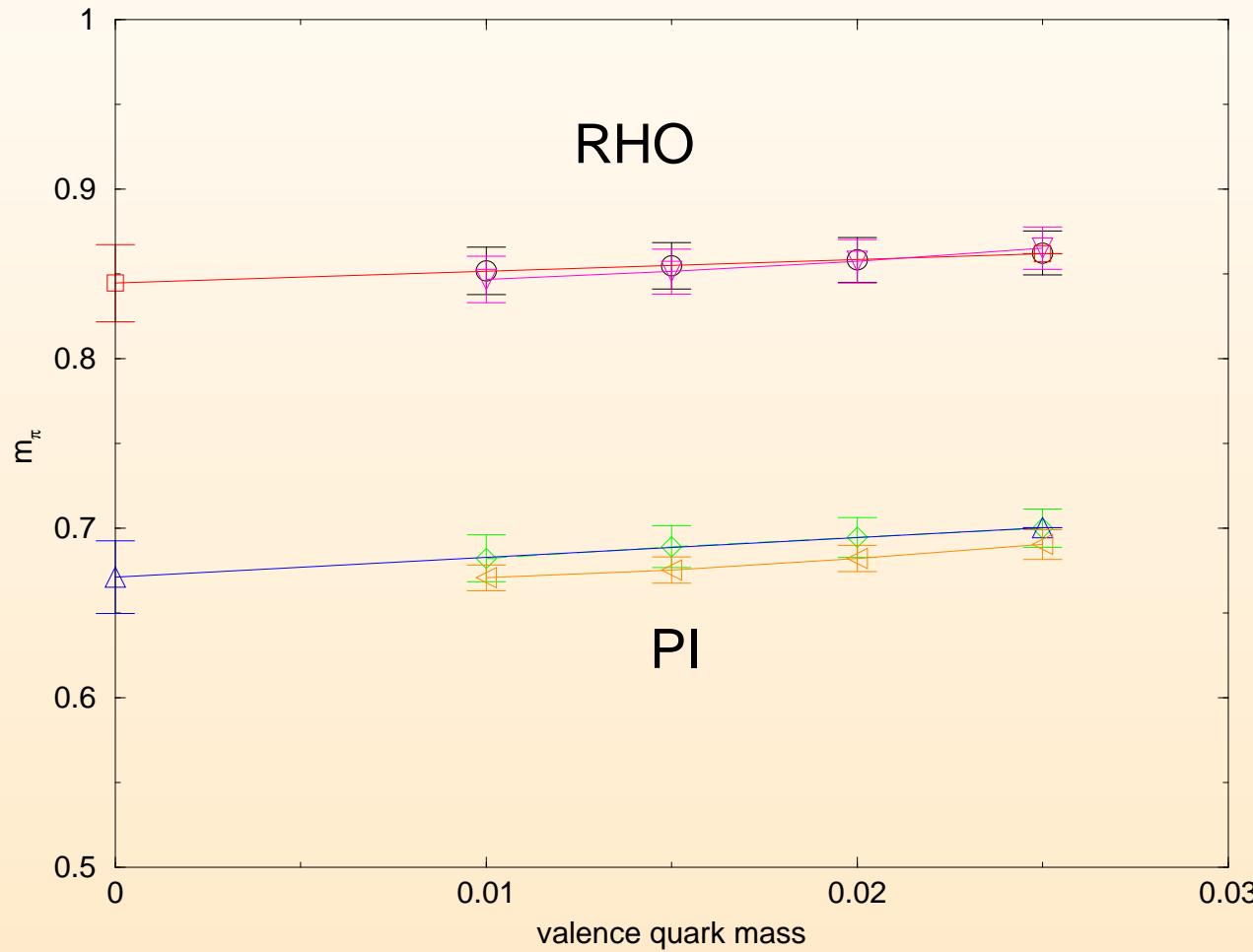


# Meson masses

$$T = a^{-1}/N_T \sim 250 \text{ MeV}$$

16x8x8x8 m=0.025 KS vs DWF

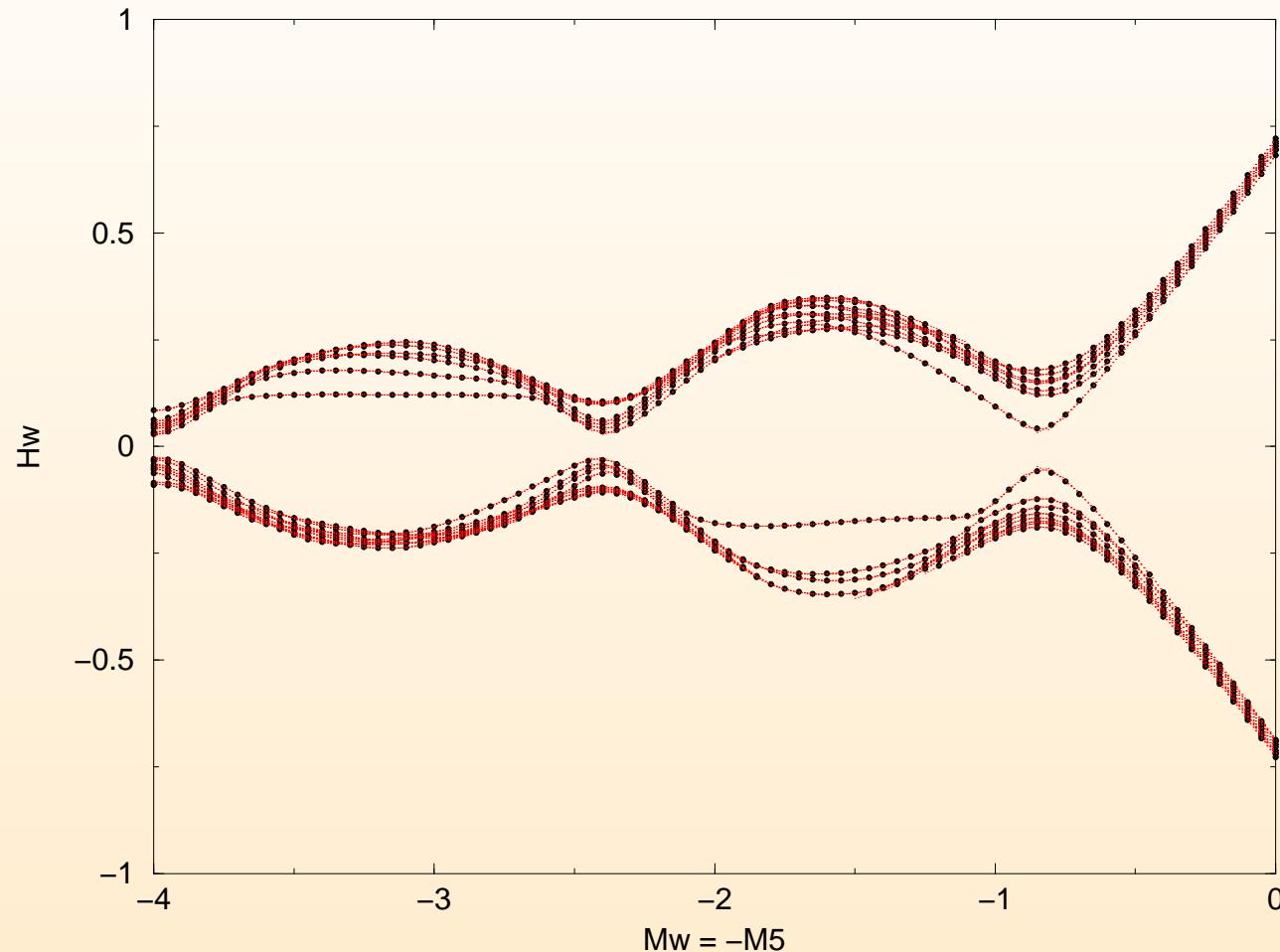
DB\_L08B080mf0025



# Spectrum Flow

Wilson Flow conf # 160

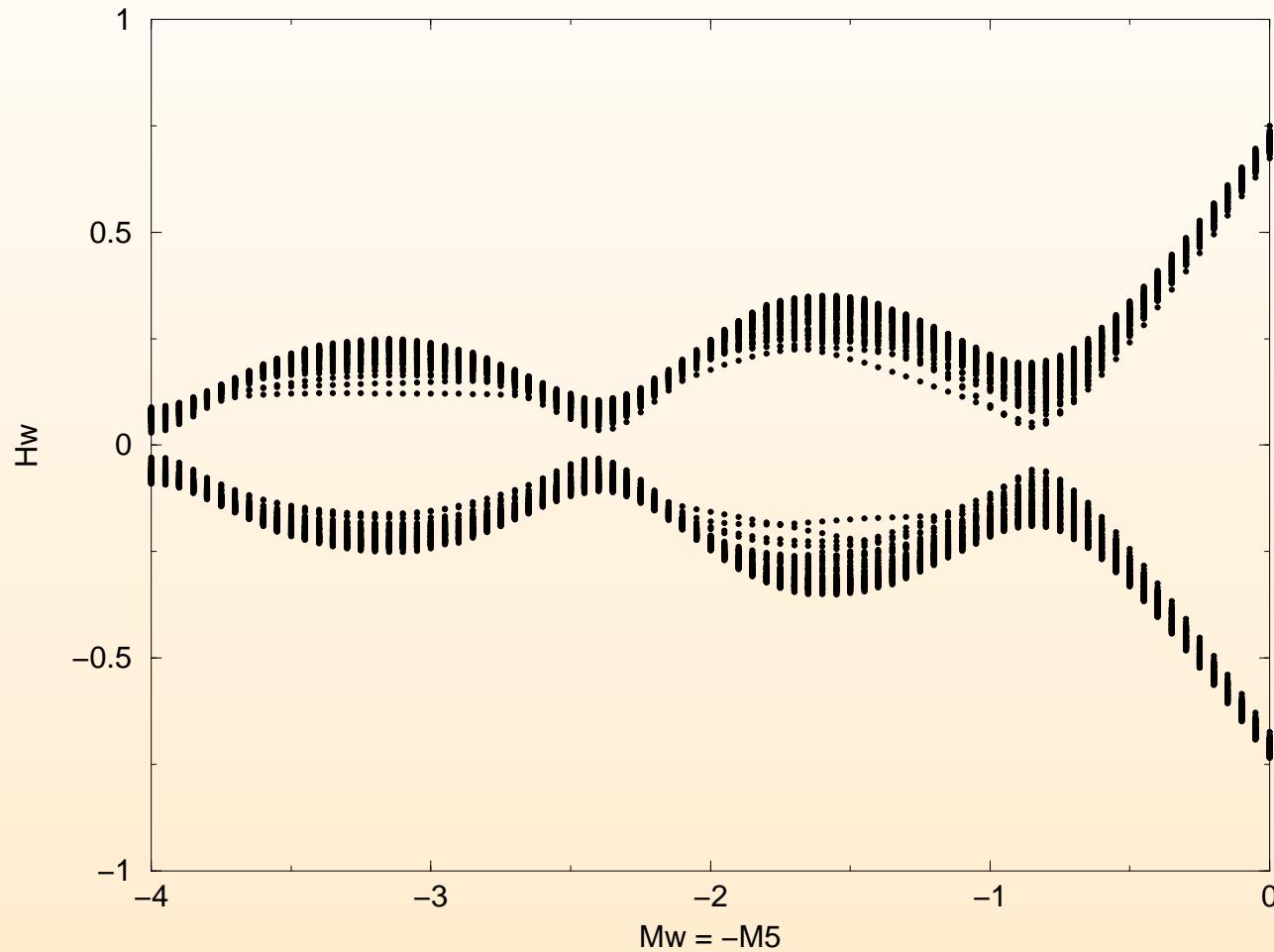
/8x16/DB\_L08B080M18mf0025/ev2wf



# Spectrum Flow

Wilson Flow conf # 100 120 140 160 180 200 220 240 80

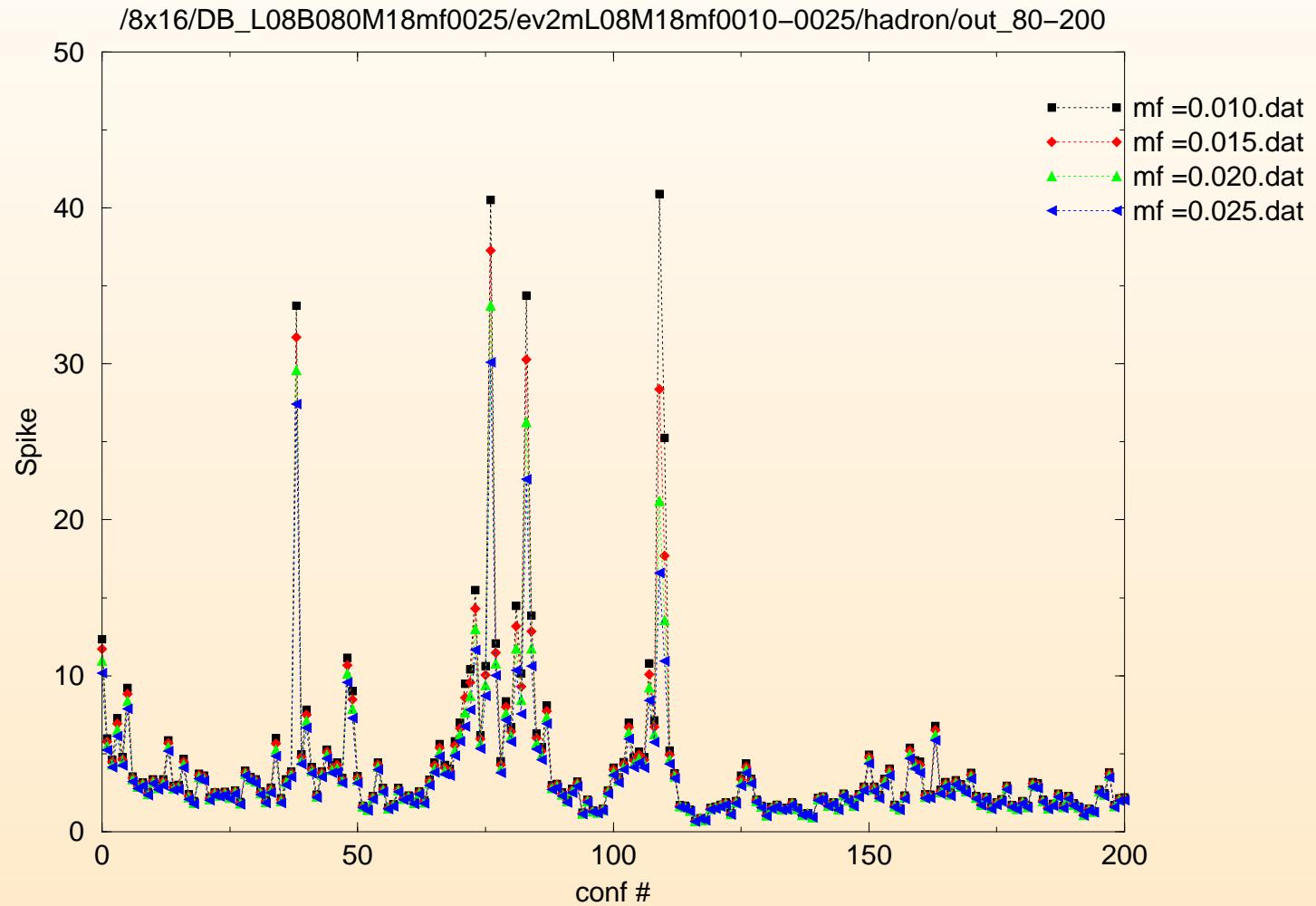
/8x16/DB\_L08B080M18mf0025/ev2wf



# history of residual mass

spikes

$\langle P \text{ J5q} \rangle$  vs Conf#

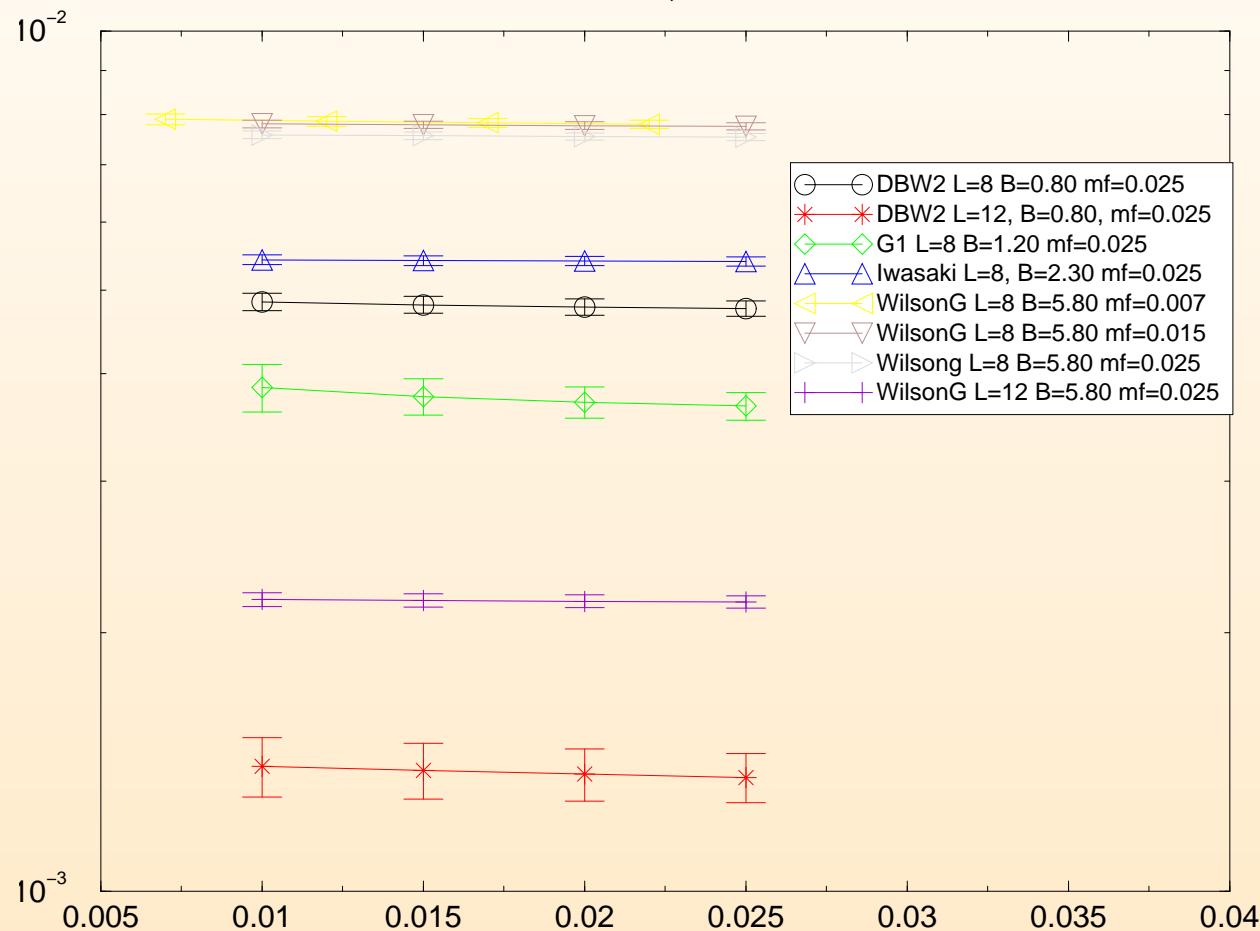


# residual masses

mildly depends on Lattice size.  $a^{-1}$  varies,

Mres vs mf

8x8x8x16, M=1.80



## chiral condensation

Axial Ward Takahashi identity

$$\begin{aligned} & \sum_x \partial_\mu \langle A_\mu(x) P(y) \rangle \\ = & \sum_x \{ 2m_f \langle P(x) P(y) \rangle + 2 \langle J_{5q}(x) P(y) \rangle - 2 \langle \bar{q} q(y) \rangle \delta_{x,y} \} \end{aligned}$$

## chiral condensation

Axial Ward Takahashi identity

$$\begin{aligned} & \sum_x \partial_\mu \langle A_\mu(x) P(y) \rangle \\ = & \sum_x \{ 2m_f \langle P(x) P(y) \rangle + 2 \langle J_{5q}(x) P(y) \rangle - 2 \langle \bar{q}q(y) \rangle \delta_{x,y} \} \\ \langle \bar{q}q(x) \rangle &= \left[ m_f + \frac{\sum_x \langle J_{5q}(x) P(y) \rangle}{\sum_x \langle P(x) P(y) \rangle} \right] \sum_x \langle P(x) P(y) \rangle \\ &= (m_f + \tilde{m}_{res}) \sum_x \langle P(x) P(y) \rangle \end{aligned}$$

## chiral condensation

Axial Ward Takahashi identity

$$\begin{aligned} & \sum_x \partial_\mu \langle A_\mu(x) P(y) \rangle \\ = & \sum_x \{ 2m_f \langle P(x) P(y) \rangle + 2 \langle J_{5q}(x) P(y) \rangle - 2 \langle \bar{q}q(y) \rangle \delta_{x,y} \} \\ \langle \bar{q}q(x) \rangle &= \left[ m_f + \frac{\sum_x \langle J_{5q}(x) P(y) \rangle}{\sum_x \langle P(x) P(y) \rangle} \right] \sum_x \langle P(x) P(y) \rangle \\ &= (m_f + \tilde{m}_{res}) \sum_x \langle P(x) P(y) \rangle \\ \sum_x \langle P(x) P(y) \rangle &< \infty \quad (T > T_c) \\ \langle \bar{q}q(x) \rangle &\rightarrow 0 \quad \text{for} \end{aligned}$$

# chiral condensation

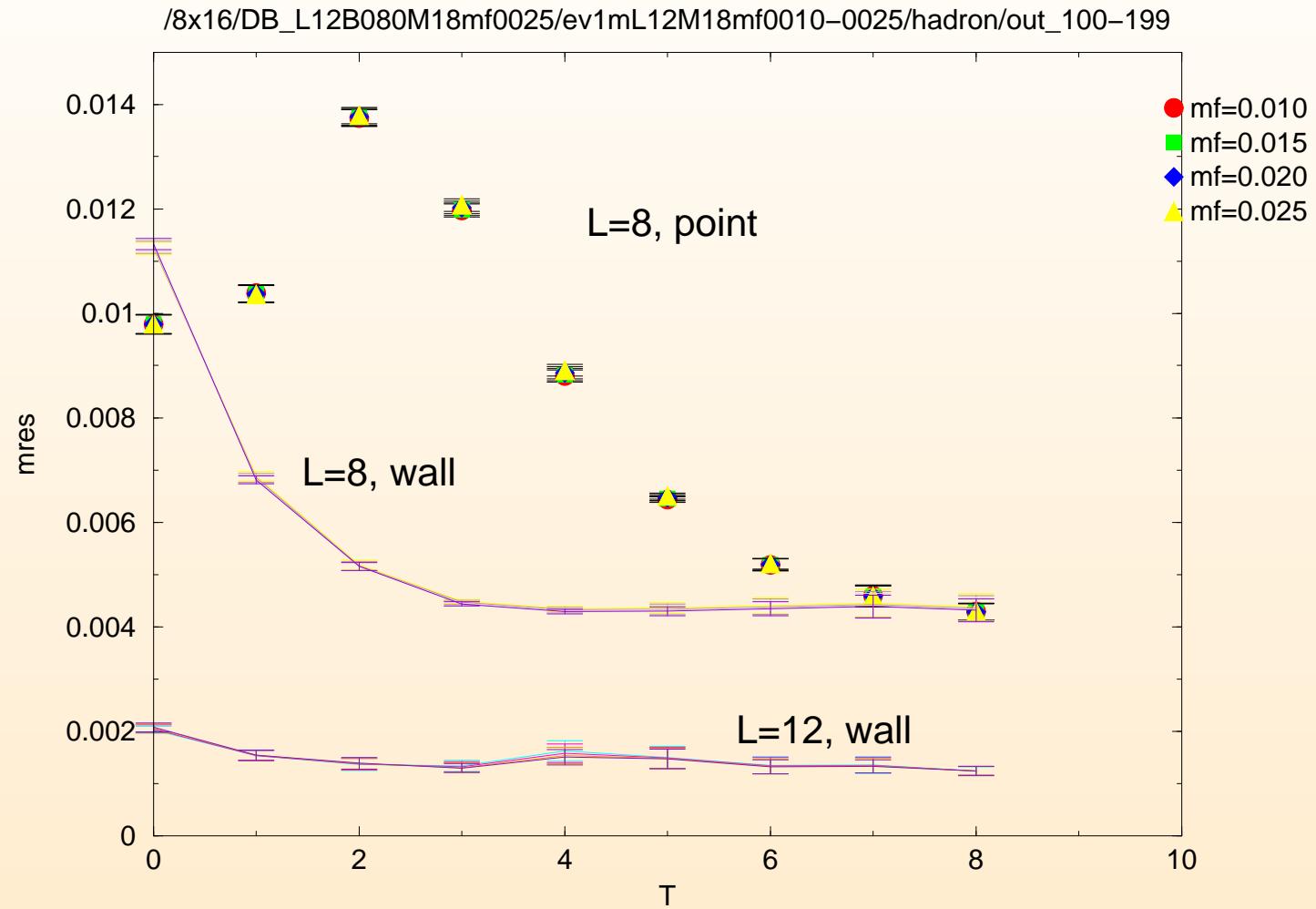
Axial Ward Takahashi identity

$$\begin{aligned}
 & \sum_x \partial_\mu \langle A_\mu(x) P(y) \rangle \\
 = & \sum_x \{ 2m_f \langle P(x) P(y) \rangle + 2 \langle J_{5q}(x) P(y) \rangle - 2 \langle \bar{q}q(y) \rangle \delta_{x,y} \} \\
 \langle \bar{q}q(x) \rangle &= \left[ m_f + \frac{\sum_x \langle J_{5q}(x) P(y) \rangle}{\sum_x \langle P(x) P(y) \rangle} \right] \sum_x \langle P(x) P(y) \rangle \\
 &= (m_f + \tilde{m}_{res}) \sum_x \langle P(x) P(y) \rangle \\
 \sum_x \langle P(x) P(y) \rangle &< \infty \quad (T > T_c) \\
 \langle \bar{q}q(x) \rangle &\rightarrow 0 \quad \text{for} \quad m_f = -\tilde{m}_{res}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{m}_{res} &= \frac{\sum_x \langle J_{5q}(x) P(y) \rangle}{\sum_x \langle P(x) P(y) \rangle} \\
 > m_{res} &= \lim_{T \rightarrow \infty} \frac{\langle J_{5q}(x) P(y) \rangle}{\langle P(x) P(y) \rangle}
 \end{aligned}$$

# $\langle J_{5q}P(T) \rangle$ : profiles of $m_{res}$

## mres profile

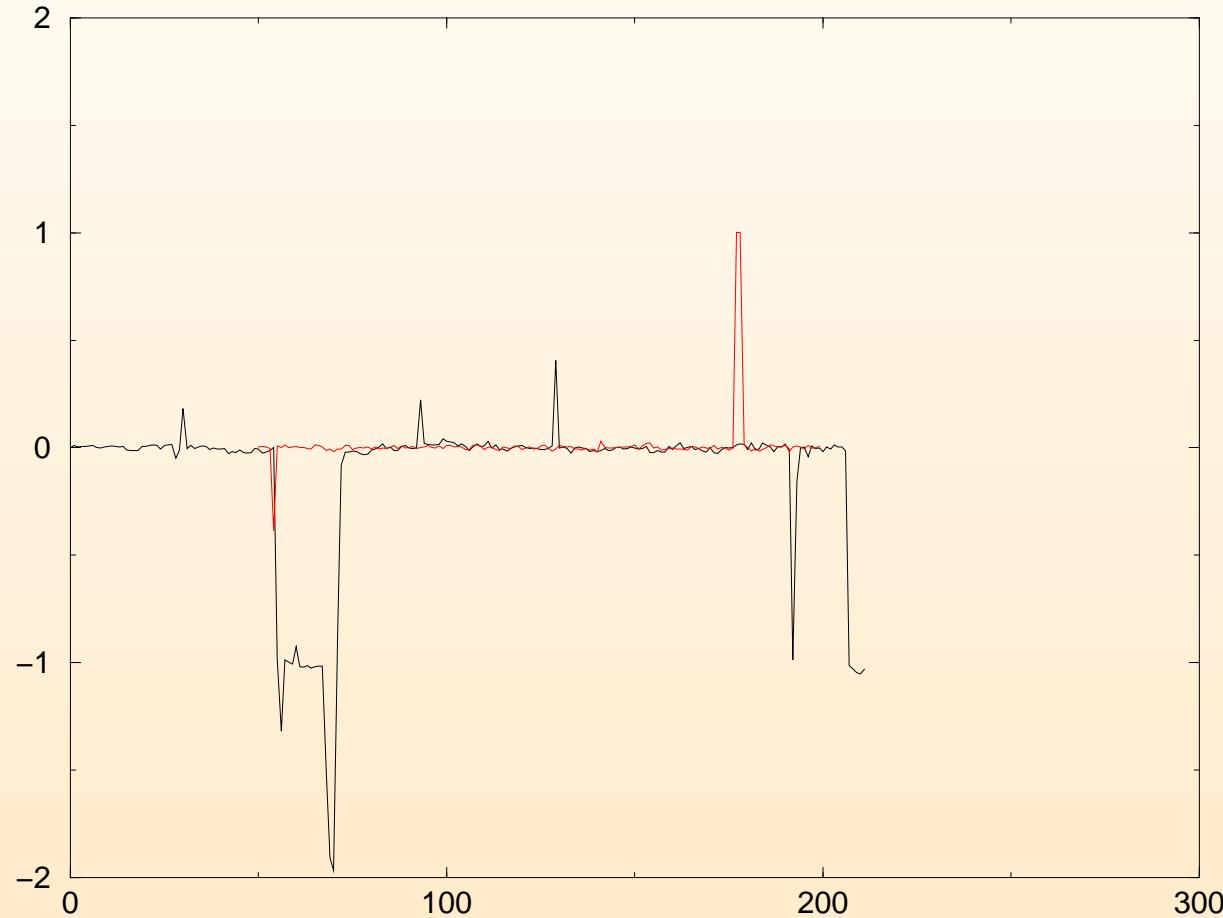


# topological charge

not always zero despite of  $\det D(m_f)$ ,  $m'_\eta \propto \langle Q_{top}^2 \rangle$

$\tilde{F}$  vs conf #

DBW2 B=0.80, mf=0.025, Ls=8, 8\*\*3x16, 16\*\*3x8



## Discussions

- At deconfined phase, on  $a^{-1} \sim 2$  GeV lattice
- $H_W$  has a rather clear gap
- 5D eigen vector is sensible (Lego plot)
- $m_{res}$  could be marginally controlled.
- quark mass dependence to chiral condensation is consistently understood.

## Discussions

- At deconfined phase, on  $a^{-1} \sim 2$  GeV lattice  $H_W$  has a rather clear gap
- 5D eigen vector is sensible (Lego plot)
- $m_{res}$  could be marginally controlled.
- quark mass dependence to chiral condensation is consistently understood.
- non zero topological charge in dynamical deconfined phase (?)

## Discussions

- At deconfined phase, on  $a^{-1} \sim 2$  GeV lattice  $H_W$  has a rather clear gap
- 5D eigen vector is sensible (Lego plot)
- $m_{res}$  could be marginally controlled.
- quark mass dependence to chiral condensation is consistently understood.
- non zero topological charge in dynamical deconfined phase (?)
- residual chiral symmetry breaking

$$S_{DWF} = S_{con} + C/P_n(L_s) e^{-\alpha L_s} O(a) + O(a^2)$$

## Discussions

- At deconfined phase, on  $a^{-1} \sim 2$  GeV lattice  $H_W$  has a rather clear gap
- 5D eigen vector is sensible (Lego plot)
- $m_{res}$  could be marginally controlled.
- quark mass dependence to chiral condensation is consistently understood.
- non zero topological charge in dynamical deconfined phase (?)
- residual chiral symmetry breaking

$$S_{DWF} = S_{con} + C/P_n(L_s) e^{-\alpha L_s} O(a) + O(a^2)$$

- profile of  $m_{res}$  shows point source'd have severer breaking,  
  ← large momentum modes has less exp suppression

$$\langle J_{5q}(x)P(y) \rangle \sim \exp(-\alpha L_s)$$

$\alpha$  has larger contribution from larger momentum modes

- Also doable for confined phase ?

- Also doable for confined phase ?
- $a^{-1}$  might depend on  $L_s$  (and  $M_5$ )

- Also doable for confined phase ?
- $a^{-1}$  might depend on  $L_s$  (and  $M_5$ )
- **needs great ideas and computational power**